Management and Innovation for a Sustainable Built Environment ISBN: 9789052693958
20 – 23 June 2011, Amsterdam, The Netherlands

THE TIME-COST ANALYSIS OF THE CONSTRUCTION PROJECT, TAKING INTO ACCOUNT RISK BASED ON EXPERT KNOWLEDGE USING FUZZY SETS

ANDRZEJ MINASOWICZ D.SC. PH.D. ENG.
Warsaw University of Technology, Poland
a.minasowicz@il.pw.edu.pl

BARTOSZ KOSTRZEWA, M.SC.
Warsaw University of Technology, Poland
bartosz.kostrzewa@gmail.com

Abstract
The analysis proposed is aimed at a detailed risk review for a given project at the stage of value engineering of the integrated value and risk management. On the basis of the cost estimate and the time schedule established, for individual groups of works, the cost or time deviations for each task are specified. Expert knowledge is used for this purpose. In order to transform the input information, it is necessary to introduce fuzzy modeling, which includes fuzzification, inference and defuzzification processes. The procedure proposed allows for automatic determination of optimistic and pessimistic project scenarios with regard to both time and cost, using simple math operators like the arithmetic average and the center of mass. In this way, we obtain the quantified risks associated with time and cost of the project, which allows for comparison of several technologies for implementation of the same project and selection of the most optimum variant.

Keywords: risk, analysis, project management, construction, fuzzy sets

INTRODUCTION

A key stage of every construction project is its implementation, or the moment of creation of the facility. This is associated with selection of the contractor, technology of performance of works, the implementation cycle. All of these aspects influence to a specific extent the two key project parameters, which are the cost and deadline of implementation.

In the article, we present the method of introducing the risk associated with a given investment at the implementation stage, using the experience and knowledge of independent experts. We used the fuzzy set theory as a tool. Fuzzy set can be presented as a set of pairs, which assigns to each element in space a degree of membership: from non-membership, through partial membership, to full membership [1]. Thus, we can see that apart from the alternative "membership – non-membership", typical for a conventional set, there are cases of partial membership here. The fuzzy set theory is a theory of classes, in which going from membership to non-membership is not incremental, like in a conventional set, but graded.

In order to transform the input information, we used fuzzy modeling, which includes fuzzification - presentation of input data (information) in form of fuzzy sets, inference-transforming several input functions into a resulting function and defuzzification- obtaining of an acute value, which reflects a given fuzzy set.

This approach towards the investment project implementation allows the investor or the general contractor to obtain knowledge on the potential changes in the time and cost of implementation.

DESCRIPTION OF THE METHOD

The example used involved the following project. The facility under concern is an apartment building, and the estimated construction cost is PLN 28 346 000. The entire project consists of seventeen groups of works, such as preparatory works, earthworks etc. Specific costs, determined by the author of the cost estimate, are associated with each group of works. These have been presented in table 1.

No.	Name	Costs
	APARTMENT BUILDING	28 346 000
1	Preparatory, background works	450 000
2	Earthworks	1 640 000
3	Raw state – underground level	2 250 000
4	Raw state – above ground part	5 300 000
5	Brickworks – external walls	950 000
6	Brickworks – internal walls	825 000
7	Roof insulation	1 320 000
8	Windows	3 120 000
9	External plaster + heat insulation	1 950 000
10	Wall lining – clinker brick	823 000
11	Interior finishing works	4 220 000
12	Passenger elevators	758 000
13	Power supply and low current installations	1 800 000
14	Sanitary installations	1 350 000
15	Building service lines	260 000
16	External works	1 150 000
17	Preparation of the facility for final acceptance	180 000

Table 1: *A tabular breakdown of costs for individual groups of works.*

The groups of works presented in table 1 constitute the time schedule of the project planned. Presented below is the Gantt chart for the project examined. The schedule below presents the planned time of implementation of individual tasks, defining the correlations between them. Apart from the sixteen groups of works, presented in table 1, the schedule includes some control points and activities that do not generate costs directly. For the needs of the method presented, we focused on analysis of deviations, associated with seventeen groups of works specified in table 1. Nevertheless, the total time of implementation of the investment is a result of twenty three tasks included in the schedule.

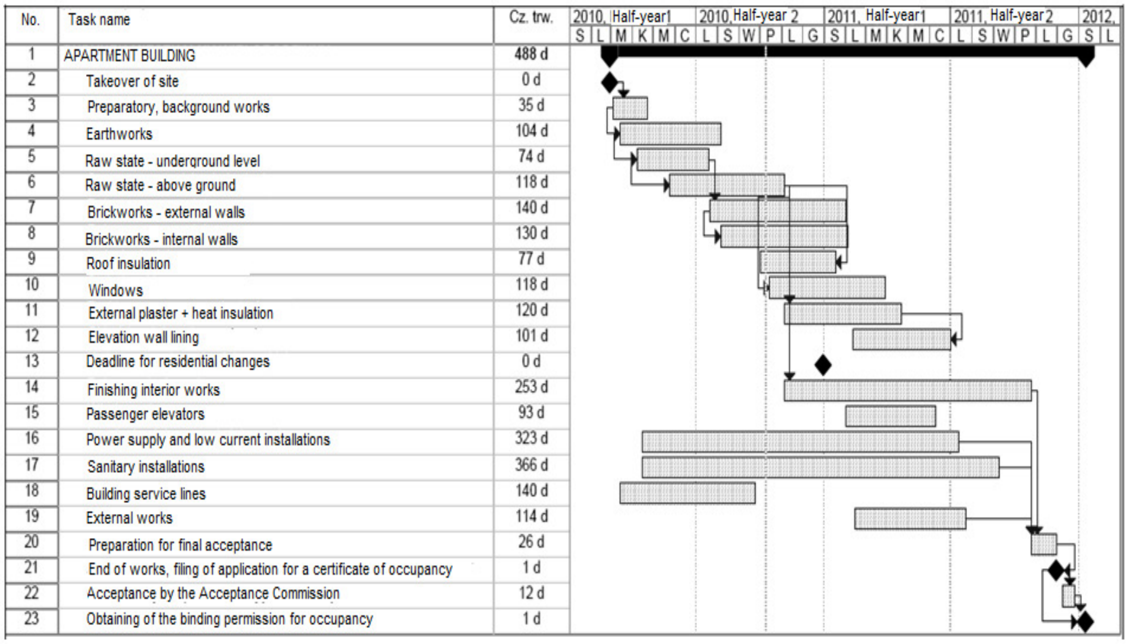

Figure 1: *Gantt chart for the project planned.*

During implementation of the project, there are various deviations from the cost or time planned. The objective of this article is to present a method of determining these prior to commencement of the project. It was assumed that information in this regard would be presented by Experts in form of 3 answers to the following questions:

What is the most probable cost/time value for task „x" and its probability (level of membership)?
What is the minimum cost/time value for task „x" and its probability (level of membership)?
What is the maximum cost/time value for task „x" and its probability (level of membership)?

For the needs of analysis, it was assumed that knowledge on deviations was obtained from 3 independent experts, using the issues associated with fuzzy modeling. At the same time, it was assumed that the probability of occurrence of deviation would be expressed as the so-called membership level. In fuzzy modeling (inference), three stages of the operating block can be distinguished: fuzzification, inference – creation of the resulting membership function, defuzzification – sharpening of the fuzzy set. The diagram has been presented in figure 2.

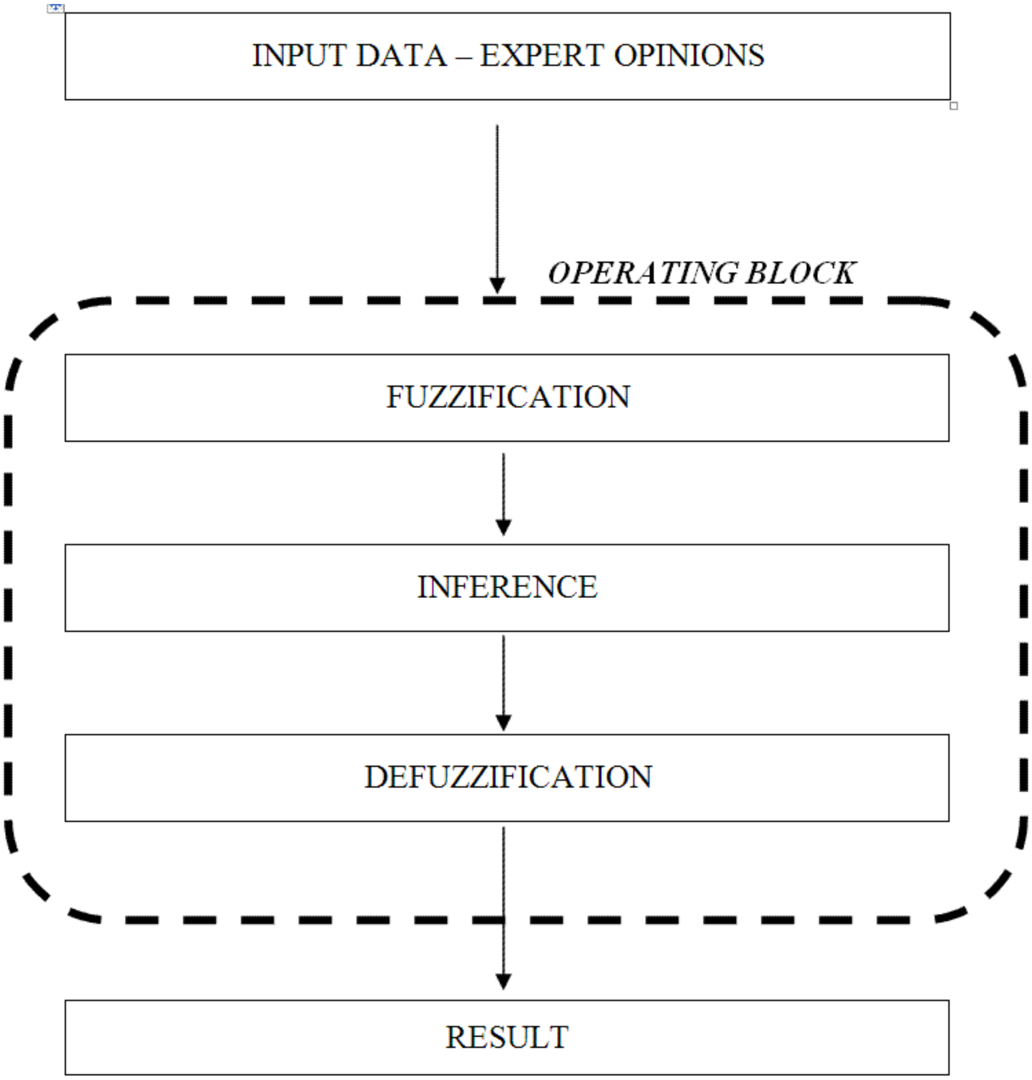

Figure 2: *A fuzzy model diagram.*

The input model data consists of answers to three questions from three experts. The first stage of the operating block is fuzzification. It is based on presentation of input data (information) in form of fuzzy sets. For the needs of this study, it was assumed that the input function of membership of the fuzzy set has the form of a linear piecewise function. The space of sets corresponding with one task of the investment has been presented below:

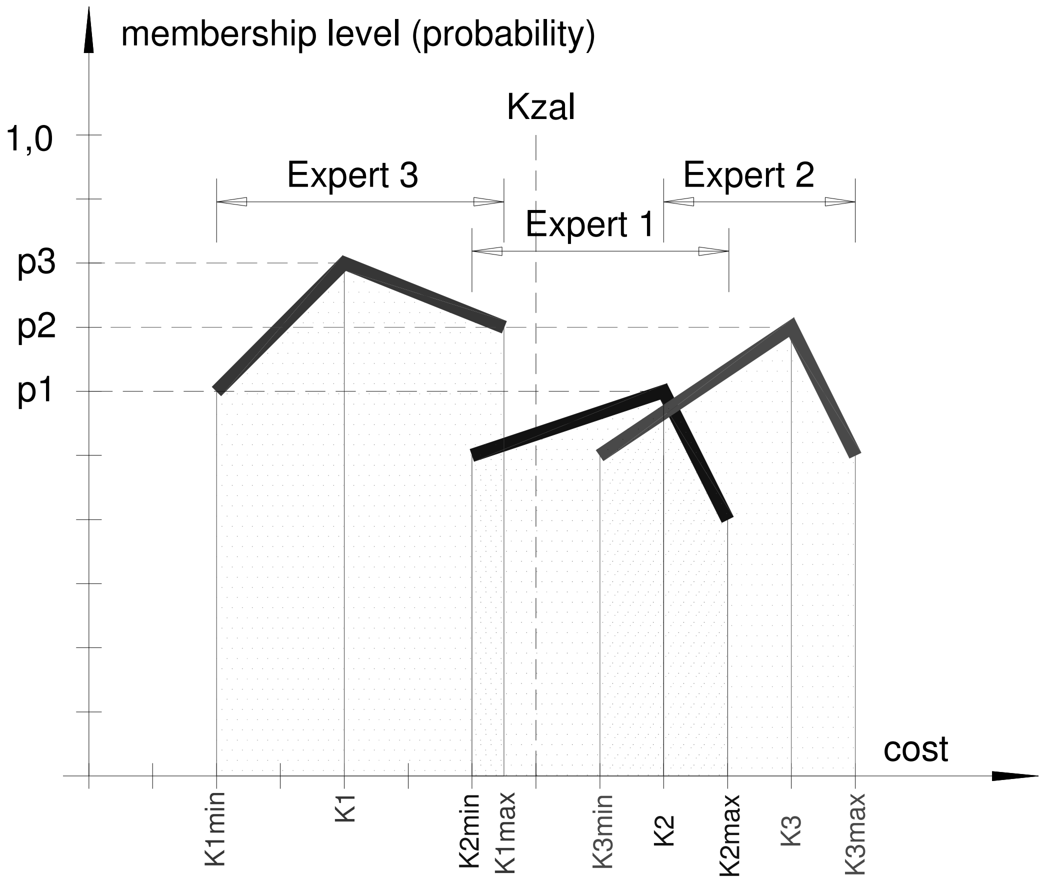

***Figure 3:** Representation of input information (expert opinions) in form of fuzzy sets*

Each of the three experts has presented the most probable cost (K1, K2, K3) and its probability (p1, p2, p3). Apart from this, the experts gave answers concerning the extreme values, that is (K1min, K2min, K3min, K1max, K2max, K3max) and the corresponding probability values (p1min, p2min, p3min, p1max, p2max, p3max). As it has been mentioned, the probability of emergence of a specific cost was expressed by the membership level. The chart also shows the Kzal, or the assumed cost. Analogically, the fuzzification process was conducted for the time of implementation of each group of works. This representation of expert knowledge presents three fuzzy sets, in which the central point is the most probable value, and the extreme values are, accordingly, the acceptable minimum and maximum values. Figure 3 depicts the fuzzy representation of input information for only one task (group of works), e.g. the preparatory/ background works. In table 2, input data on cost deviations for all investment tasks can be found.

No.	Kzal	Kemin1	pemin1	Ke1	pe1	Kemax1	pemax1
1	450 000	315 000	0,65	562 500	0,90	585 000	0,85
2	1 640 000	1 230 000	0,65	1 476 000	0,70	1 558 000	0,60
3	2 250 000	1 462 500	0,60	1 800 000	0,90	1 912 500	0,90
4	5 300 000	3 445 000	0,60	5 565 000	0,95	6 095 000	0,70
5	950 000	1 045 000	0,90	1 092 500	0,95	1 140 000	0,90
6	825 000	618 750	0,65	825 000	0,70	948 750	0,65
7	1 320 000	792 000	0,80	1 056 000	0,90	1 188 000	0,60
8	3 120 000	2 964 000	0,75	3 432 000	0,75	3 744 000	0,65
9	1 950 000	1 755 000	0,75	2 145 000	0,80	2 242 500	0,60
10	823 000	658 400	0,65	1 028 750	0,75	1 069 900	0,65
11	4 220 000	3 798 000	0,60	4 642 000	0,75	5 064 000	0,75
12	758 000	492 700	0,75	795 900	0,85	871 700	0,70
13	1 800 000	1 440 000	0,80	1 620 000	0,85	2 070 000	0,60
14	1 350 000	1 080 000	0,70	1 147 500	0,70	1 417 500	0,65
15	260 000	182 000	0,65	208 000	0,80	260 000	0,80
16	1 150 000	977 500	0,65	1 380 000	0,80	1 495 000	0,70
17	180 000	117 000	0,65	144 000	0,85	171 000	0,75

No.	Kzal	Kemin2	pemin2	Ke2	pe2	Kemax2	pemax2
1	450 000	270 000	0,70	495 000	0,80	517 500	0,75
2	1 640 000	1 394 000	0,60	2 050 000	0,80	2 132 000	0,70
3	2 250 000	1 462 500	0,70	1 687 500	0,85	2 025 000	0,60
4	5 300 000	3 975 000	0,70	4 505 000	0,90	5 565 000	0,90
5	950 000	1 092 500	0,65	1 187 500	0,85	1 235 000	0,75
6	825 000	536 250	0,90	948 750	0,95	990 000	0,80
7	1 320 000	1 254 000	0,75	1 584 000	0,95	1 650 000	0,75
8	3 120 000	2 184 000	0,85	3 120 000	0,90	3 588 000	0,80

9	1 950 000	1 267 500	0,95	2 145 000	0,95	2 340 000	0,95
10	823 000	534 950	0,60	699 550	0,85	781 850	0,60
11	4 220 000	3 587 000	0,70	4 220 000	0,90	4 642 000	0,70
12	758 000	871 700	0,60	947 500	0,80	985 400	0,60
13	1 800 000	1 440 000	0,65	1 530 000	0,85	1 710 000	0,65
14	1 350 000	1 080 000	0,70	1 147 500	0,70	1 417 500	0,65
15	260 000	182 000	0,65	208 000	0,80	260 000	0,80
16	1 150 000	977 500	0,65	1 380 000	0,80	1 495 000	0,70
17	180 000	117 000	0,65	144 000	0,85	171 000	0,75

No.	Kzal	Kemin3	pemin3	Ke3	pe3	Kemax3	pemax3
1	450 000	292 500	0,75	315 000	0,85	450 000	0,75
2	1 640 000	1 312 000	0,75	1 804 000	0,75	1 886 000	0,70
3	2 250 000	1 800 000	0,70	2 137 500	0,80	2 250 000	0,75
4	5 300 000	3 710 000	0,70	3 975 000	0,85	5 035 000	0,60
5	950 000	855 000	0,65	997 500	0,90	1 045 000	0,70
6	825 000	866 250	0,75	948 750	0,75	1 031 250	0,60
7	1 320 000	924 000	0,60	990 000	0,80	1 188 000	0,65
8	3 120 000	2 028 000	0,75	2 184 000	0,75	2 340 000	0,75
9	1 950 000	2 047 500	0,60	2 145 000	0,90	2 340 000	0,80
10	823 000	699 550	0,80	864 150	0,90	946 450	0,85
11	4 220 000	2 743 000	0,60	3 587 000	0,80	4 009 000	0,75
12	758 000	568 500	0,65	606 400	0,90	758 000	0,85
13	1 800 000	1 440 000	0,75	1 530 000	0,75	1 890 000	0,70
14	1 350 000	945 000	0,70	1 755 000	0,90	1 822 500	0,60
15	260 000	169 000	0,75	221 000	0,75	286 000	0,70
16	1 150 000	1 035 000	0,75	1 380 000	0,80	1 437 500	0,60
17	180 000	153 000	0,60	171 000	0,80	189 000	0,70

Table 2: *A breakdown of input cost data for all investment tasks.*

In the fuzzy sets theory, Figure 3 simply presents 3 fuzzy sets. This is input information. For further use of the knowledge of 3 experts, information presented by them was consolidated to one set representing the cost (time) deviations for a given task. In other words, the so-called resulting membership function was presented. The process transforming several input functions into a resulting function in fuzzy modeling is known as inference. There are many operators used to determine the resulting membership function. In this article, we used the arithmetic average operator. The value of the resulting membership function is equal to the average level of membership of each input set, which can be recorded according to following formula:

$$f_{wynik}(k) = \frac{\sum_{i=1}^{n} f_i(k)}{n},$$

where:

$f_{wynik}(k)$ – the resulting membership function (function determining the probability of occurrence of events),

k – function argument – cost or time

n – number of experts (number of fuzzy sets), n=3,

i=1, 2, …, n.

Upon the assumptions made, the formula can be recorded as follows:

$$f_{wynik}(k) = \frac{f_{EKS1}(k) + f_{EKS2}(k) + f_{EKS3}(k)}{3},$$

where:

$f_{EKS1}(k)$ – membership function determined on the basis of data from expert 1,

$f_{EKS2}(k)$ – membership function determined on the basis of data from expert 2,

$f_{EKS3}(k)$ – membership function determined on the basis of data from expert 3.

Using the MS-Excel spreadsheet, the resulting function was established for each investment task, presenting the potential risk information.

Figures 4 and 5 below present the operation of inference, using as an example the first group of works in the project or „Preparatory, background works”

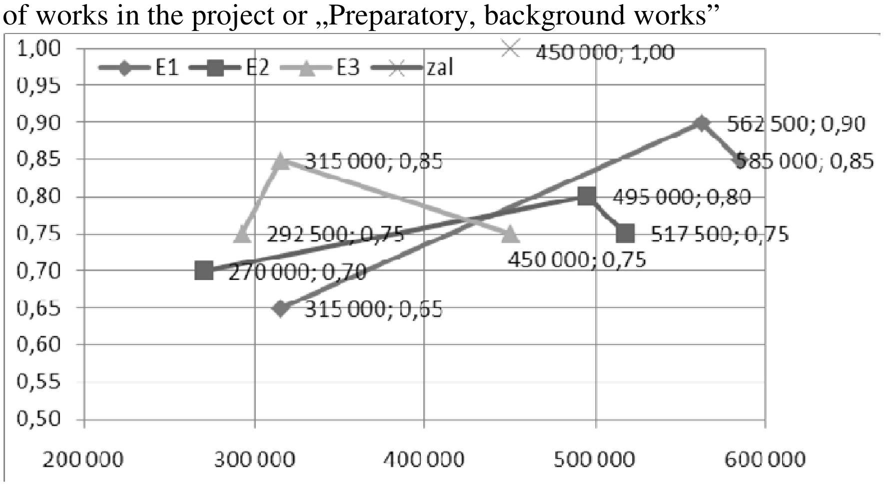

Figure 4: *Input functions – opinions of three Experts concerning risk for the task „Preparatory, background works”.*

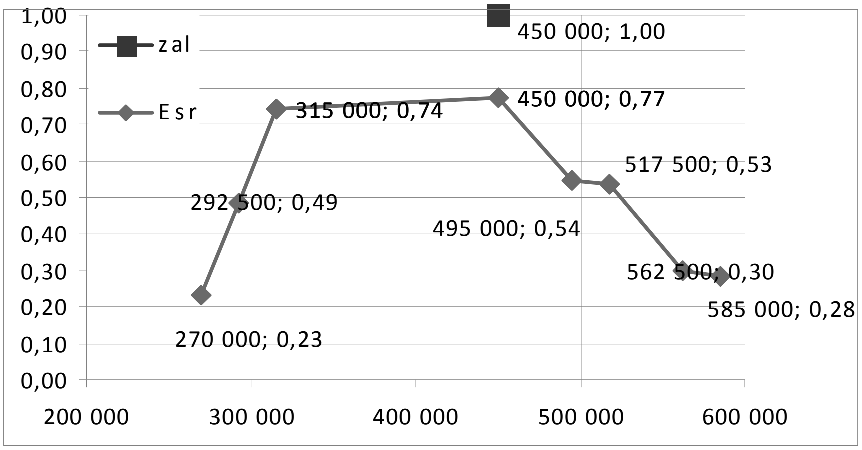

Figure 5: *Resulting risk function for task „Preparatory, background works".*

The defuzzification process leads to obtaining of an acute value, which reflects a given fuzzy set. Having the resulting function of the expert opinion, we established two points, dividing the set of results into the optimistic and pessimistic part. These points were established using the center of mass method. The acute value was calculated on the basis of following formula:

$$k_{wyn} = \frac{\int k \cdot u_{wyn}(k)dk}{\int u_{wyn}(k)dk},$$

where:

k_{wyn} – the acute costs value (input value),

$u_{wyn}(k)$ – the resulting membership function,

k – the cost (argument) of the membership function.

In the presented case, the resulting membership function is in form of a broken line. The center of mass of the broken line, e.g. ABCD, was determined by replacing each line section with a material point, placed in the middle of the section, of the mass equal to the section length [2]. The coordinates of the center of mass of the broken ABCD were determined on the basis of following formulas. According to Figure 6 symbols d_1, d_2, d_3 represent lengths of sections AB, BC, CD, and $S_1(k_1,u_1)$, $S_2(k_2,u_2)$, $S_3(k_3,u_3)$ are centers of these sections.

$$k_0 = \frac{d_1k_1 + d_2k_2 + d_1k_3}{d_1 + d_2 + d_3},$$

$$u_0 = \frac{d_1u_1 + d_2u_2 + d_1u_3}{d_1 + d_2 + d_3}.$$

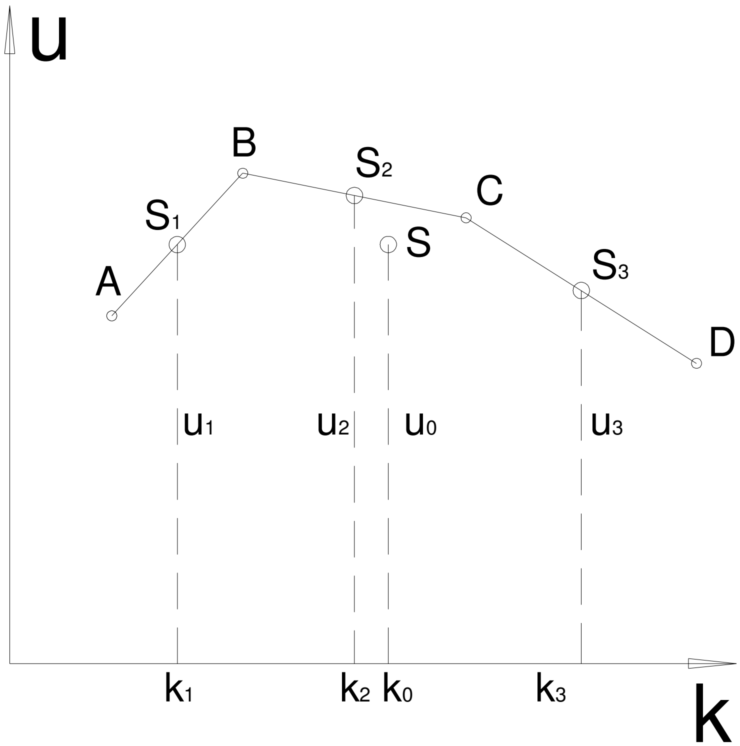

Figure 6: *Graphic representation of determination of the center of mass of a broken line.*

On the basis of the above formulas and using the Excel spreadsheet, for each group of works, optimistic and pessimistic centers of mass were established. For the first group of costs, the "Preparatory, background works", were presented in Figure 7.

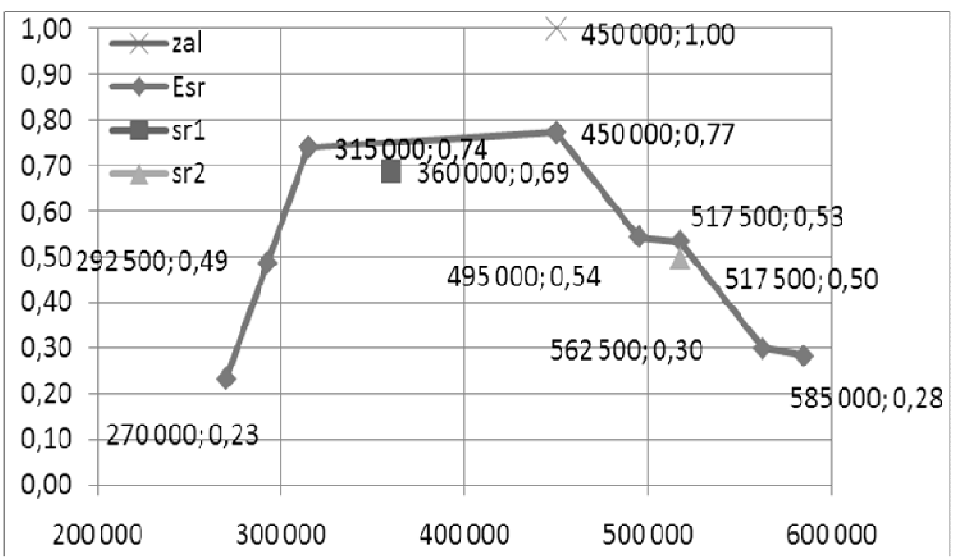

Figure 7: *Graphic representation of establishing of the center of mass for the first group of costs of „Preparatory, background works".*

The same operations, that is, fuzzification, inference, defuzzification were conducted for all groups of investment costs. The results obtained were presented in table 3 for cost values and in table 4 for time values.

No.	Name	Est cost	Opt cost	popt	Pes cost	ppes
	APARTMENT BUILDING	28 346 000	22 648 750	0,53	31 438 350	0,46
1	Preparatory, background works	450 000	360 000	0,69	517 500	0,50
2	Earthworks	1 640 000	1 394 000	0,57	1 886 000	0,41
3	Raw state – underground level	2 250 000	1 631 250	0,53	2 081 250	0,40
4	Raw state – above ground part	5 300 000	3 975 000	0,60	5 565 000	0,54
5	Brickworks –external walls	950 000	950 000	0,30	1 163 750	0,41
6	Brickworks – internal walls	825 000	701 250	0,52	1 419 000	0,29
7	Roof insulation	1 320 000	990 000	0,45	1 419 000	0,29
8	Windows	3 120 000	2 496 000	0,52	3 432 000	0,47
9	External plaster, heat insulation	1 950 000	1 706 250	0,54	2 242 500	0,75
10	Wall lining – clinker brick	823 000	658 400	0,52	946 450	0,43
11	Interior finishing works	4 220 000	3 376 000	0,45	4 642 000	0,44
12	Passenger elevators	758 000	625 350	0,50	890 650	0,33
13	Power supply and low current installations	1 800 000	1 485 000	0,77	1 845 000	0,42
14	Sanitary installations	1 350 000	1 012 500	0,55	1 586 250	0,48
15	Building service lines	260 000	195 000	0,67	286 000	0,51
16	External works	1 150 000	948 750	0,47	1 322 500	0,48
17	Preparation of the facility for final acceptance	180 000	144 000	0,37	193 500	0,46

Table 3: *A breakdown of pessimistic and optimistic cost values with probability.*

No.	Name	Time	Opt t	popt	Pes t	ppes
	APARTMENT BUILDING	488	415	0,48	539	0,47
1	Preparatory, background works	35	29	0,57	39	0,48
2	Earthworks	104	96	0,65	130	0,54
3	Raw state – underground level	74	59	0,75	83	0,53
4	Raw state – above ground part	118	86	0,47	124	0,23
5	Brickworks –external walls	140	115	0,27	161	0,46
6	Brickworks – internal walls	130	104	0,48	75	0,51
7	Roof insulation	77	58	0,44	75	0,51
8	Windows	118	91	0,52	127	0,48
9	External plaster + heat insulation	120	81	0,67	117	0,56
10	Wall lining – clinker brick	101	83	0,58	116	0,43
11	Interior finishing works	253	215	0,63	291	0,62
12	Passenger elevators	93	70	0,54	102	0,34
13	Power supply and low current installations	323	234	0,46	323	0,38
14	Sanitary installations	366	302	0,29	421	0,48
15	Building service lines	140	119	0,50	158	0,52
16	External works	114	97	0,51	120	0,39
17	Preparation of the facility for final acceptance	26	23	0,50	33	0,48

Table 4: *A breakdown of pessimistic and optimistic time values with probability.*

In the analysis presented, the measure of risk applied was the ratio of the value of deviation to its probability. Thanks to the above analysis of each investment task, it is possible to establish the summarized deviations associated with the entire project. Figures 8 and 9 present the resulting estimated, pessimistic and optimistic values of cost and time. They were based on aggregation of individual values for investment tasks, using the MS-Project software to create 3 scenarios of the task implementation: optimistic, estimated and pessimistic. The resulting probability of summarized values has been established as a weighted average.

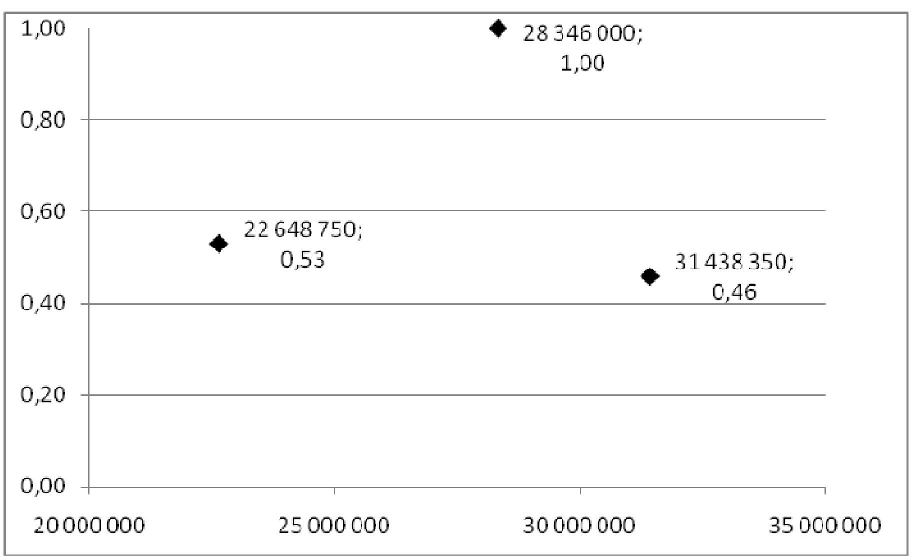

Figure 8: *Graphic representation of risk for the investment cost*

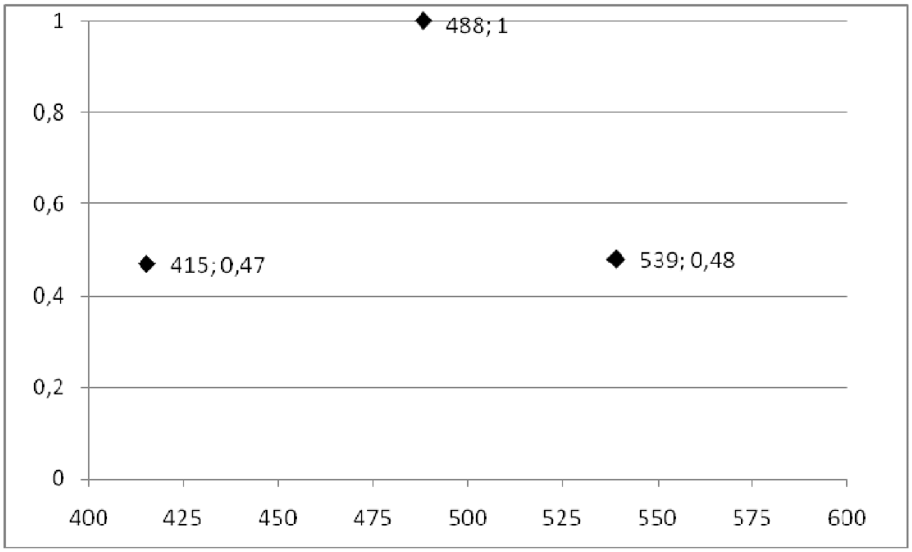

Figure 9: *Graphic representation of risk for the investment time.*

Thanks to this representation of the cost and time of implementation, the general project risk, as function of probability and deviation value was determined on the basis of following formulas:

$$R_k^{pes} = \frac{(K_{pes} - K_{zal})/K_{zal}}{1 - p_{pes}} = \frac{(31438000 - 28346000)/28346000}{1 - 0{,}46} = 0{,}202$$

$$R_k^{opt} = \frac{(K_{zal} - K_{opt})/K_{zal}}{1 - p_{opt}} = \frac{(28346000 - 22648750)/28346000}{1 - 0{,}53} = 0{,}428$$

On the basis of the same formulas and according to figure 9 it is possible to determine risk values for the investment time.

CONCLUSIONS

The method presented provides the general contractor or the investor with knowledge on the potential deviations, cost and time associated risks. This method uses expert knowledge associated with individual stages of works, which are very diversified. Expert knowledge has been used separately for each stage, at the same time obtaining the risk associated with the entire project.

On the basis of quantification of risk, investor or contractor obtain the possibility of responding quickly to unexpected scenarios. Thanks to this analysis, commencing the investment, they are aware of the potential threats associated with a failure to meet the deadline or exceeding of the budget planned. Moreover it is possible to compare several possible projects on the basis of cost and time deviation and its probabilities.

The analysis proposed makes it possible to control the project further during its implementation. It is possible to use the data obtained to control the project e.g. using the earned value method [3]. Thanks to time and cost analysis, at the level of individual tasks, it was possible to determine cash flows in the time function for different variants.

In this way, at any time during the project life cycle, the investor or the general contractor is able to determine whether the threshold values of the earned value method indicators have not been exceeded. At the same time, at the stage preceding decision-making, the investor is able to determine the possible risk (variance) of the assumed implementation cost or time.

Having the knowledge on the time and cost variances so far, the general contractor will find it easier to plan the financing of the project, without exposing the project to additional problems, associated with delayed payments.

THE PROCEDURE ALGORITHM – A BLOCK DIAGRAM

Presented below are the general rules of the procedure in form of a block diagram.

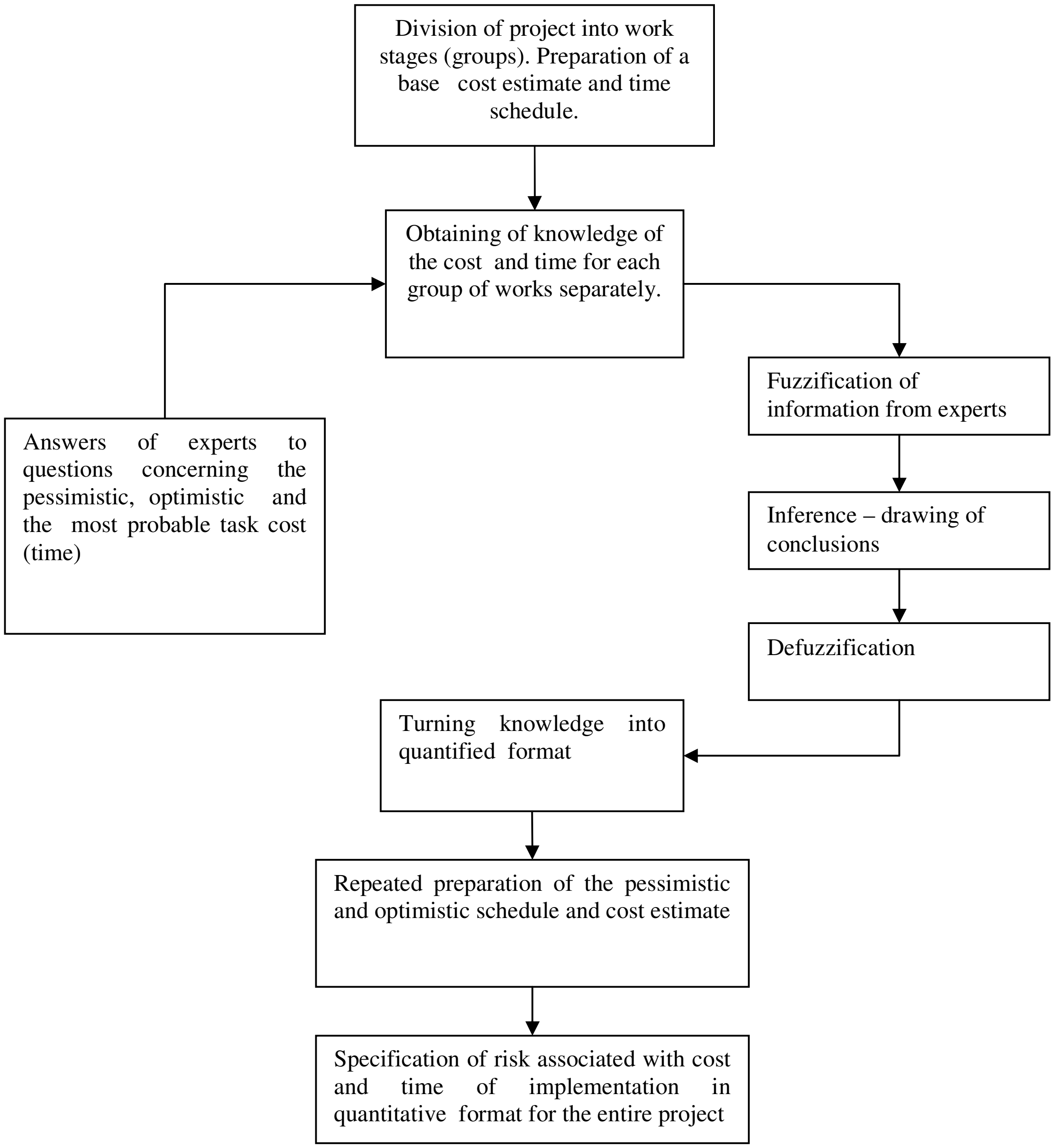

Figure 9: *A block diagram of time-cost analysis of the project planned, taking into account the risk, on the basis of expert knowledge, using fuzzy sets.*
`

LITERATURE

[1] Kacprzyk J., *Zbiory rozmyte w analizie systemowej*. Państwowe Wydawnictwo Naukowe, Warszawa 1986.
[2] Banach S., *Mechanika. Monografie matematyczne.* Tom VIII Warszawa, Lwów, Wilno, 1938.

[3] Webb A., *Wartość wypracowana w praktyce.* PROED 2008
[4] Buckley J., Siler W., *Fuzzy Expert Systems and Fuzzy Reasoning*, John Wiley & Sons, 2004.
[5] Chapman C., Ward S., *Project risk management*, Wiley, 2003.
[6] Kacprzyk J., *Multistage fuzzy control,* John Wiley and Sons, Chichester, 1997.
[7] McCaffer R., *Modern Construction Management,* Blackwell Science, 2006.